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A Perturbation Solution for the Laminar Boundary Layer on a Continuous Moving Surface

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SUMMARY

A perturbation solution is presented for the nonsimilar boundary layer flow past a moving surface with constant wall velocity. A zero pressure gradiant is assumed and the tangential velocity and stagnation enthalpy profiles are prescribed at the initial station. The eigen values obtained for the first order perturbation solution are integers and the eigen functions are the derivatives of the error function. The same Green's function is obtained for all the higher order perturbation equations, and the higher order perturbation solutions are given in integral form in terms of the Green's function. The analyses include both the momentum equation and the energy equation which is uncoupled with the momentum equation, and which is subjected to a nonsimilar velocity flow field.

1. Introduction

The similar solutions for the laminar boundary layer formed on a continuous moving surface have been investigated by several authors [1–3]. The momentum equation governing such flow is the Blasius equation with nonvanishing velocity on the moving surface. Recently, Mager [4] considered the problem in which a similar boundary layer with zero velocity on the surface, which is described by the Blasius solution, is suddenly accelerated by a belt moving in the plane of the wall at a constant speed in the downstream direction. Solutions of the momentum equation at positions very near and very far from the station where the belt is first encountered were obtained. The nonsimilar solutions of the boundary layer equation with zero pressure gradient were obtained as a perturbation from the Blasius solution by Libby and Fox [5, 6] for a flow past a stationary surface. They obtained the discrete eigen values and the corresponding eigen functions of the perturbation equations.

In this paper, perturbation solutions are presented for the problem in which a fluid with zero pressure gradient flows past a surface moving with a constant speed which differs from that of the free stream. For this problem the tangential velocity of arbitrary profile which is close to the Blasius solution is prescribed at the initial station. The present problem is similar to the initial value problem of boundary layer flow treated in refs. [5] and [6] with the exception that the tangential velocity on the surface is zero in refs. [5] and [6]. The present problem is different from that of ref. [4] since the sudden acceleration of the boundary layer from the initial station will not be considered here.

Apparently, the present problem can be treated by the method of refs. [5] and [6]. In the formulation of ref. [5] the tangential velocity function $f_\xi(s, \xi)$ is written as

$$f_\xi(s, \xi) = f_0'(\xi) + f_{1\xi}(s, \xi) + f_{2\xi}(s, \xi) + \ldots \tag{1.1}$$

with the independent variables ξ and s defined as

$$\xi = \rho_e u_e (\tfrac{1}{2} s^{\frac{1}{2}}) \int_0^y (\rho/\rho_e) dy \quad \text{and} \quad s = \int_0^x \rho_e u_e u_e dx \tag{1.2}$$

where ρ is density; u, velocity; μ, viscosity; y, the normal coordinate; x, the coordinate in stream wise direction, and the subscript e represents properties at the free stream condition. The independent variable ξ, instead of the Levy-Lees variable $\eta = \sqrt{2}\xi$ as used in ref. [5] is adopted here for convenience in the analyses. The basic solution $f_0(\xi)$ is the Blasius solution which satisfies

$$f_0''' + 2f_0 f_0'' = 0 \tag{1.3}$$

subject to the boundary conditions

$$f_0(0) = 0 \; ; \quad f_0'(0) = u_0/u_e = q \; ; \quad f_0'(\infty) = 1 \tag{1.4}$$

with u_0, the velocity of the moving surface, and the notation (′) represents derivative with respect to ξ. The Blasius solution defined here is different from the conventional Blasius solution in which $f_0'(0) = 0$. Nevertheless the eigen values and eigen functions for a given value of q can be obtained in the same manner as ref. [5] by numerically integrating the first order perturbation equation and imposing the exponential decay of the velocity function $f_\xi(s, \xi)$ as $\xi \to \infty$. Since the Blasius solution is a function of q, the eigen values obtained by this method will also depend on q. This will lead to a different set of eigen values and eigen functions for each value of q if the method of ref. [5] is employed. It is the purpose of this paper to present a perturbation solution in which the eigen values and the eigen functions are independent of the Blasius solutions, and thus independent of q. In the present formulation the similar solution $f_0(\xi)$ as well as the nonsimilar solution is expanded in power series of the perturbation parameter α. The eigen values obtained are integers and the perturbation solutions are given in terms of derivatives of the error function.

The perturbation solutions of the momentum equation will be given in the next section followed by the perturbation solutions of the energy equation for an adiabatic wall and a constant enthalpy wall subject to a nonsimilar flow. The Green's functions of the higher order perturbation equations can be shown to be identical. Therefore all the higher order perturbation solutions can be obtained in terms of this Green's function.

2. Momentum Equation

Consider the nonsimilar boundary layer flow past a surface moving at a constant speed u_0 as shown in Fig. 1. A zero pressure gradient is assumed, and the tangential velocity profile is

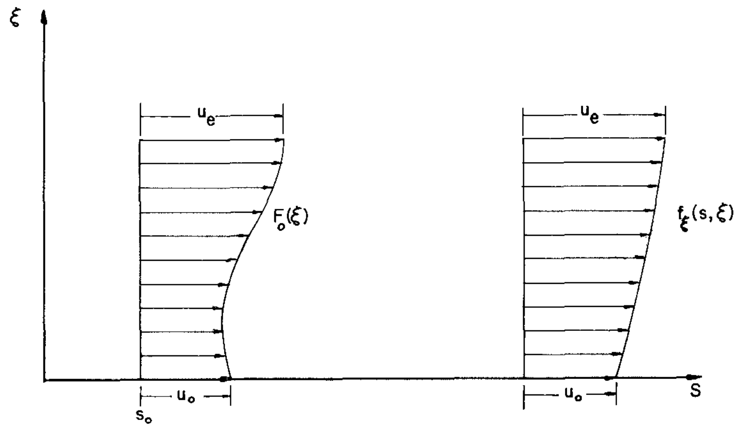

Figure 1. Uniform flow past a continuous moving surface with a given initial velocity profile at $s = s_0$.

prescribed at the initial station $s = s_0 \neq 0$. The momentum equation governing such flow is the nonsimilar boundary layer equation [7],

$$f_{\xi\xi\xi} + 2ff_{\xi\xi} - 4s(f_\xi f_{\xi s} - f_{\xi\xi} f_s) = 0 \tag{2.1}$$

where $f(s, \xi)$ is the modified stream function.

The initial condition at $s = s_0$ is

$$f_\xi(s_0, \xi) = F_0(\xi) \tag{2.2a}$$

and the boundary conditions at $\xi=0$ and $\xi=\infty$ are

$$f_\xi(s,0)=q, \quad f_\xi(s,\infty)=1, \quad f(s,0)=0 \tag{2.2b}$$

The tangential velocity profile $F_0(\xi)$ given at the initial station $s=s_0$ is assumed to satisfy $F_0(0)=q$ and $F_0(\infty)=1$, so that sudden acceleration of the boundary layer as treated in ref. [4] is excluded from our consideration. To obtain a solution for $f(s,\xi)$ let

$$f(s,\xi)=f_0(\xi)-\alpha w(s,\xi) \tag{2.3}$$

where $f_0(\xi)$ is the Blasius solution defined in Eqs. (1.3) with the boundary conditions Eq. (1.4). The perturbation parameter is defined to be $\alpha=1-q$, with the value of q assumed to be $2>q>0$ and $q\neq 1$. The nonsimilar solution $w(s,\xi)$ as the perturbations from the Blasius solution is considered, and the initial and boundary conditions are then $w(s_0,\xi)=[f_0'(\xi)-F_0(\xi)]/\alpha$ and $w_\xi(s,0)=w_\xi(s,\infty)=w(s,0)=0$. The nonsimilar solution $w(s,\xi)$ as well as the Blasius solution is then expanded in power series of α as follows,

$$w(s,\xi)=w^{(0)}(s,\xi)+\alpha w^{(1)}(s,\xi)+\alpha^2 w^{(2)}(s,\xi)+\dots \tag{2.4}$$

$$f_0(\xi)=\xi-\alpha w_0(\xi)-\alpha^2 w_1(\xi)-\alpha^3 w_2(\xi)+\dots \tag{2.5}$$

Substitution of Eqs. (2.3)–(2.5) into Eq. (2.1) and equating the same powers of α yields the perturbation equations for the Blasius solution,

$$\mathscr{L}_1 w_0'=0 \tag{2.6a}$$

$$\mathscr{L}_1 w_n'=2\sum_{k=0}^{n-1} w_k w_{n-k-1}'' \qquad n=1,2,3\dots \tag{2.6b}$$

Where the linear operator $\mathscr{L}_1$ is defined as

$$\mathscr{L}_1=\frac{d^2}{d\xi^2}+2\xi\frac{d}{d\xi}$$

The boundary conditions for Eq. (2.6a) and (2.6b) are, respectively,

$$w_0'(0)=1, \quad w_0(0)=w_0'(\infty)=0 \tag{2.7a}$$

and

$$w_n(0)=w'_n(0)=w_n'(\infty)=0 \qquad n=1,2,3\dots \tag{2.7b}$$

The perturbation equations governing $w^{(n)}(s,\xi)$ are

$$\mathscr{L}_2 w_\xi^{(0)}=0 \tag{2.9a}$$

$$\mathscr{L}_2 w_\xi^{(n)}(s,\xi)=R_n(s,\xi) \qquad n=1,2,3\dots \tag{2.9b}$$

Where

$$\mathscr{L}_2=\frac{\partial^2}{\partial\xi^2}+2\xi\frac{\partial}{\partial\xi}-4s\frac{\partial}{\partial s}$$

$$\begin{aligned}R_n(s,\xi)=2\sum_{k=0}^{n-1}\{&w^{(k)}w_{n-k-1}''+w_k w_{\xi\xi}^{(n-k-1)}\\&+w^{(k)}w_{\xi\xi}^{(n-k-1)}+2s[-w_k' w_{s\xi}^{(n-k-1)}+w_\xi^{(k)}w_{s\xi}^{(n-k-1)}\\&+w_k'' w_s^{(n-k-1)}+w_{\xi\xi}^{(k)}w_s^{(n-k-1)}]\} \qquad n=1,2,3\dots\end{aligned} \tag{2.9c}$$

The initial and boundary conditions for Eqs. (2.9a) and (2.9b) are

$$w_\xi^{(0)}(s_0,\xi)=F(\xi)=[f_0'(\xi)-F_0(\xi)]/\alpha$$

$$w_\xi^{(0)}(s,0)=w_\xi^{(0)}(s,\infty)=w^{(0)}(s,0)=0 \tag{2.10a}$$

$$w_\xi^{(n)}(s_0,\xi)=w_\xi^{(n)}(s,0)=w_\xi^{(n)}(s,\infty)=w^{(n)}(s,0)=0 \qquad n=1,2,3\dots \tag{2.10b}$$

The solutions w_0 and w_1 in the set of Eqs. (2.6) subject to boundary conditions (2.7) can be obtained readily. The results are [12]

$$w_0 = \xi \operatorname{erfc} \xi - \pi^{-\frac{1}{2}}(e^{-\xi^2} - 1) \tag{2.11}$$

$$w_1 = \pi^{-\frac{1}{2}}(1-\pi^{-1})(1-e^{-\xi^2}) + (\tfrac{3}{2}\pi^{\frac{1}{2}})e^{-\xi^2} \operatorname{erf} \xi - (2/\pi)^{\frac{1}{2}} \operatorname{erf} 2^{\frac{1}{2}} \xi \tag{2.12}$$
$$- (\tfrac{1}{2})\xi \operatorname{erf} \xi \operatorname{erfc} \xi - (1/\pi)\xi \operatorname{erfc} \xi + \pi^{-\frac{1}{2}} \operatorname{erf} \xi$$

where erfc ξ is the complementary error function. Subsequent solutions in the sequences given by Eq. (2.6b) can be found by the following integrations:

$$w_n' = 2 \int_\xi^\infty e^{-\bar{\xi}^2} d\bar{\xi} \int_{\bar{\xi}}^\infty \sum_{k=0}^{n-1} w_k(\xi') w_{n-k-1}(\xi') e^{\xi'^2} d\xi' + K_n \operatorname{erfc} \xi \qquad n = 2, 3, 4, \ldots \tag{2.13}$$

where K_n is a constant determined from the condition $w_n'(0)=0$, and is given by

$$K_n = -2 \int_0^\infty e^{-\bar{\xi}^2} d\bar{\xi} \int_{\bar{\xi}}^\infty \sum_{k=0}^{n-1} w_k(\xi') w_{n-k-1}''(\xi') e^{\xi'^2} d\xi' \qquad n = 2, 3, 4, \ldots$$

The solution $w_n(\xi)$ is then obtained by integrating $w_n'(\xi)$,

$$w_n(\xi) = \int_0^\xi w_n'(\bar{\xi}) d\bar{\xi}\ , \qquad n = 2, 3, 4 \ldots \tag{2.14}$$

The approximate value of $f_0''(0)$ can be obtained from Eqs. (2.5) and (2.11 ~ 2.14). The result is

$$f_0''(0) = (2\alpha/\pi^{\frac{1}{2}})(1 - \alpha\pi^{-1}) + \alpha^3 w_3''(0) + \ldots \tag{2.15}$$

values of $f_0''(0)$ computed from Eq. (2.15) are shown in Fig. 2 together with that obtained from the numerical solutions of Eq. (1.3) subject to the boundary conditions Eq. (1.4) for various values of $f'(0)=q$ in the range of $0<q<1$.

In Eq. (1.3) the boundary conditions are given at $\xi=0$ and $\xi=\infty$, and a trial-and-error process is usually required for the numerical solutions. For the numerical method adopted here the trial-and-error process is eliminated by transforming the two point boundary value problem to an initial value problem [8]. Some of the numerical values of $f''(0)$ obtained from Eqs. (1.3)

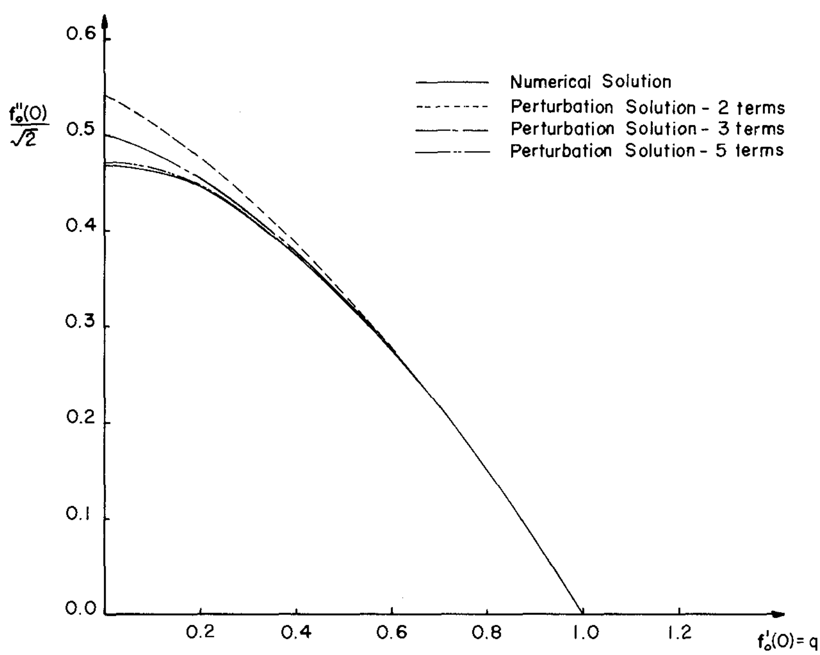

Figure 2. Comparison of exact solution and perturbation solutions for the Blasius solution.

and (1.4) also may be found in ref. [2]. It can be seen from Fig. 2 that for a small value of α the approximate values of $f_0''(0)$ given in Eq. (2.15) give a good approximation to the exact numerical solutions. As is to be expected, for a larger value of α more terms of the perturbation solutions will be needed to obtain an accurate result.

By means of separation of variables, the solution $w_\xi^{(0)}(s, \xi)$ of Eq. (2.9a) will be represented by a series of products as follows [10],

$$w_\xi^{(0)}(s, \xi) = \sum_k A_k (s/s_0)^{-k/2} \phi_k(\xi) \tag{2.16}$$

The differential equation for $\phi_k(\xi)$ is

$$\phi_k'' + 2\xi\phi_k' + 2k\phi_k = 0 \tag{2.17}$$

and the boundary conditions are

$$\phi_k(0) = 0 \quad \text{and} \quad \phi_k(\infty) \to 0 \tag{2.18}$$

With this weak boundary condition at $\xi \to \infty$, the eigen values k have a continuous spectrum $k > 0$. Since the exponential decay of the tangential velocity as $\xi \to \infty$ is essential for boundary layer behavior [9], it is necessary to impose a stronger condition,

$$\phi_k(\xi) = o(\xi^{-N}) \quad \text{for any} \quad N > 0 \tag{2.19}$$

With Eq. (2.19) instead of the second equation in Eqs. (2.18) the eigen values are discrete and are even integers 2, 4, 6, ... and the solutions ϕ_k are the kth derivative of the error function [11]. The constant A_k is related to the initial data by the orthogonal condition of ϕ_k with $\exp(\xi^2)$ as the weight function,

$$A_k = [\pi^{\frac{1}{2}}/2^k(k-1)!] \int_0^\infty F(\xi) \exp(\xi^2)\, \phi_k(\xi) d\xi \qquad k = 2, 4, 6 \ldots \tag{2.20}$$

Subsequent solutions, for $n > 1$, given by Eqs. (2.9b) and (2.9c) subject to homogeneous initial and boundary conditions are nontrivial due to the non-homogeneous term $R_n(s, \xi)$, and can be obtained in terms of a Green's function $G(s, \xi, \tilde{s}, \tilde{\xi})$ defined in the same manner as ref. [5] by

$$G_{\xi\xi} + 2\,\xi G_\xi - 4s\, G_s = \delta(\xi - \tilde{\xi})\,\delta(s - \tilde{s}) \tag{2.21}$$

Using separation of variables, a series solution for the Green's function may be constructed. The result is

$$\begin{aligned} G(s, \xi, \tilde{s}, \tilde{\xi}) &= -\sum_{k=2,4,6,\ldots} [\pi^{\frac{1}{2}}/2^{k+3}(k-1)!]\, \phi_k(\tilde{\xi})\, \phi_k(\xi)\, e^{\tilde{\xi}^2}\, \tilde{s}^{(k/2)-1}/s^{k/2} && \text{for } s > \tilde{s} \\ &= 0 \qquad \text{for } \tilde{s} > s \geqq s_0 && \end{aligned} \tag{2.22}$$

The solution $w_\xi^{(n)}$ is then

$$w_\xi^{(n)}(s, \xi) = \int_0^\infty \int_{s_0}^s G(s, \xi, \tilde{s}, \tilde{\xi})\, R_n(\tilde{s}, \tilde{\xi})\, d\tilde{s}\, d\tilde{\xi} \qquad n = 1, 2, 3 \ldots \tag{2.23}$$

The skin friction function evaluated at the moving surface $f_{\xi\xi}(s, 0)$ is then given by the following expression:

$$\begin{aligned} f_{\xi\xi}(s, 0) &= -\sum_{n=0}^\infty \alpha^{n+1} [w_n''(0) + w_{\xi\xi}^{(n)}(s, 0)] \\ &= f_0''(0) - \sum_{k=2,4,6\ldots} \Big\{ \alpha A_k (s/s_0)^{k/2} \phi_k'(0) - \sum_{n=1}^\infty \alpha^{n+1} [\pi^{\frac{1}{2}}/2^{k+3}(k-1)!]\, \phi_k'(0) \times \\ &\qquad \times \int_0^\infty \int_{s_0}^s [\phi_k(\tilde{\xi}) \exp(\tilde{\xi}^2)\, R_n(\tilde{s}, \tilde{\xi})\, s^{(k/2)-1}/s^{k/2}]\, d\tilde{s}\, d\tilde{\xi} \Big\} \end{aligned} \tag{2.24}$$

3. Energy Equation

The energy equation for a laminar boundary layer with a uniform free stream, the ratio of density viscosity product $C \equiv \rho\mu/\rho_e\mu_e = 1$ and the Prandtl number $Pr = 1$ can be written in terms of the dependent variables $g(s, \xi)$, the stagnation enthalpy ratio defined by $g = h_s/h_{se}$, and $f(s, \xi)$, the modified stream function, by

$$g_{\xi\xi} + 2fg_\xi - 4s(f_\xi g_s - f_s g_\xi) = 0 \tag{3.1}$$

We consider the nonsimilar flow given by Eq. (2.1) subject to the initial and boundary conditions (2.2a) and (2.2b). The energy equation (3.1) is uncoupled with the momentum equation, and is a linear equation. To find a perturbation solution let

$$g(s, \xi) = g^{(0)}(s, \xi) + \alpha g^{(1)}(s, \xi) + \alpha^2 g^{(2)}(s, \xi) + \dots \tag{3.2}$$

substitution of Eqs. (2.3)–(2.5) and (3.2) into Eq. (3.1) yields

$$\mathscr{L}_2 g^{(0)}(s, \xi) = 0 \tag{3.3a}$$

$$\mathscr{L}_2 g^{(n)}(s, \xi) = H_n(s, \xi) \qquad n = 1, 2, 3, \dots \tag{3.3b}$$

where

$$H_n(s, \xi) = \sum_{k=0}^{n-1} \left[2\, \tilde{w}^{(k)} g_\xi^{(n-k-1)} - 4s(\tilde{w}_\xi^{(k)} g_s^{(n-k-1)} - \tilde{w}_s^{(k)} g_\xi^{(n-k-1)})\right] \qquad n = 1, 2, 3, \dots \tag{3.4}$$

with

$$\tilde{w}^{(k)} = w_k(\xi) + w^{(k)}(s, \xi) \tag{3.5}$$

The zeroth order solution in Eq. (3.3a) is the exact solution of the energy equation for the flow, $f_\xi(\xi) = 1$. The higher order solutions in Eq. (3.3b) are the solutions of the energy equation for flow deviated from $f_\xi = 1$.

Two basic problems each corresponding to $C = 1$, $Pr = 1$, and with a nonsimilar velocity profile $f(s, \xi)$ obtained from Eq. (2.1), (2.2a) and (2.2b) will be presented. The initial and boundary conditions of interest in the first problem to be considered are

$$g(s_0, \xi) = G_1(\xi), \quad g(s, 0) = g_w = \text{const.}, \quad g(s, \infty) = 1 \tag{3.6}$$

This problem represents a boundary layer flow, which has a nonsimilar velocity profile flowing on a continuous moving surface of constant enthalpy, having arbitrary distribution of stagnation enthalpy profile $G_1(\xi)$ at $s = s_0 \neq 0$. To find a solution let

$$g^{(0)}(s, \xi) = 1 + (g_w - 1)\,\text{erfc}(\xi) + g_1^{(0)}(s, \xi) \tag{3.7}$$

substitution of Eq. (3.7) into Eq. (3.3a) yields the equation for $g_1^{(0)}(s, \xi)$,

$$\mathscr{L}_2 g_1^{(0)}(s, \xi) = 0 \tag{3.8}$$

with the initial and boundary conditions

$$g_1^{(0)}(s_0, \xi) = G_1(\xi) - 1 - (g_w - 1)\,\text{erfc}(\xi) \tag{3.9a}$$

$$g_1^{(0)}(s, 0) = g_1^{(0)}(s, \infty) = 0 \tag{3.9b}$$

Again, when the stronger condition $g_1^{(0)} = o(\xi^{-N})$ for any N as $\xi \to \infty$ is imposed, the solution for $g_1^{(0)}$ is

$$g_1^{(0)}(s, \xi) = \sum_{k=2,4,6\dots} B_k (s/s_0)^{-k/2} \phi_k(\xi) \tag{3.10}$$

where

$$B_k = [\pi^{\frac{1}{2}}/2^k(k-1)!] \int_0^\infty [G_1(\xi) - 1 - (g_w - 1)\,\text{erfc}(\xi)] \exp(\xi^2)\phi_k(\xi)\,d\xi \qquad k = 2, 4, 6, \dots \tag{3.11}$$

Higher order perturbation solutions in Eq. (3.3b) can be obtained in the same manner as in the momentum equation,

$$g^{(n)}(s,\xi)=\int_0^\infty\int_{s_0}^s G(s,\xi,\tilde{s},\tilde{\xi})H_n(\tilde{s},\tilde{\xi})\,d\tilde{s}\,d\tilde{\xi} \qquad n=1,2,3\ldots \tag{3.12}$$

where the Green's function $G(s,\xi,\tilde{s},\tilde{\xi})$ is given in Eq. (2.22).

The second problem to be considered is a problem corresponding to a boundary layer which has a nonsimilar velocity profile, which flows on an adiabatic continuous moving surface, having an arbitrary distribution of stagnation enthalpy profile $G_2(\xi)$ at $s=s_0\neq 0$. The initial and boundary conditions for this problem are

$$g(s_0,\xi)=G_2(\xi), \quad g_\xi(s,0)=0, \quad g(s,\infty)=1 \tag{3.13}$$

The solution may be found in the same manner as for the first problem; let

$$g^{(0)}(s,\xi)=1+g_2^{(0)}(s,\xi) \tag{3.14}$$

Substitution of Eq. (3.14) into Eq. (3.3a) yields again $\mathscr{L}_2 g_2^{(0)}(s,\xi)=0$ with the initial and boundary condition $g_2^{(0)}(s,\xi)=G_2(\xi)-1$, and $g_{2\xi}^{(0)}(s,0)=g_2^{(0)}(s,\infty)=0$. The solution $g_2^{(0)}(s,\xi)$ is then identical to $g_1^{(0)}(s,\xi)$ given in Eq. (3.10) except the eigenvalues for this case are 1, 3, 5 ... and the constant B_k given by

$$B_k=[\pi^{\frac{1}{2}}/2^k(k-1)!]\int_0^\infty [G_2(\xi)-1]\exp(\xi^2)\phi_k(\xi)d\xi \qquad k=1,3,5,\ldots \tag{3.15}$$

The higher order solutions are then determined from Eq. (3.12) as in the first problem.

4. Concluding Remark

Flows which are described by perturbations about the Blasius solution with constant tangential velocity at the wall have been considered. The first order perturbation solutions are the set of eigenfunctions which are the derivatives of the error function, and the eigenvalues are integers. The higher order perturbation solutions are given in terms of the Green's function and can be obtained by means of quadrature.

It has been assumed that $0<|\alpha|<1$ in the formulation of the perturbation solutions, however for the case $\alpha=1$, i.e. the flow over a stationary surface, the present method still valid due to the fact that the perturbation solution of the Blasius solution in Eq. (2.5) is a sequence of successively decreasing series even for $\alpha=1$. It is recognized that the present method when applied to the case with $\alpha=1$ will converge to the exact solution slower than the method of ref. [5], and the higher order perturbation solutions will be needed for obtaining accurate results. In the case $\alpha=0$ i.e. $f_0(\xi)=\xi$, by limiting process as $\alpha\to 0$ it can be shown that the present formulation is identical to that of refs. [5] and [6]. It is to be noted that in the present formulations, as well as that of ref. [5], in order the linearized theory be valid, the initial data prescribed at the initial station must be only slightly different from the Blasius solution. In particular if $F_0(\xi)=f_0'(\xi)$ then the nonsimilar solution $w(s,\xi)$ vanishes; the flow is similar with $f(s,\xi)=f_0(\xi)$. Although no numerical example has been attempted in this paper, the nonsimilar effects due to the deviation from the Blasius solution can be obtained by straightforward integrations of the functions related to the error function.

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